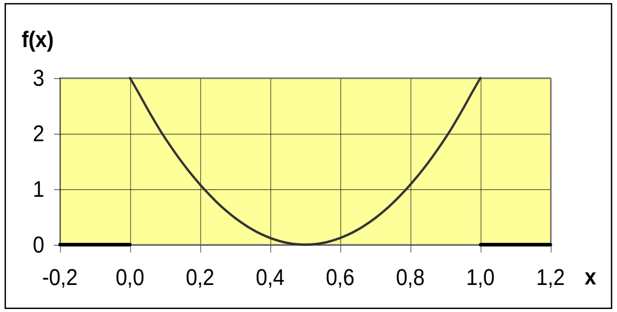
Aufgabe 6.4

a) $f_X(x) = \mathbf{12\,x^2 - 12\,x + 3}$

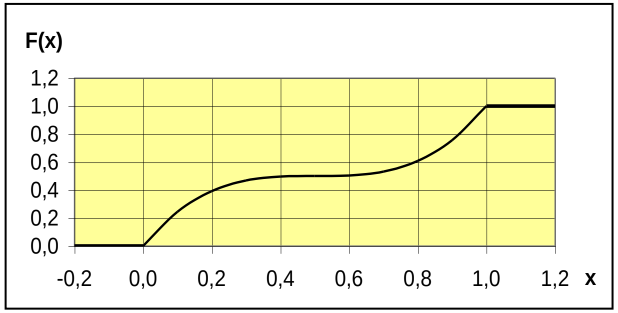
b) **Wertetabelle**

x	0	0,2	0,5	0,8	1
$f_X(x)$	3	1,080	0	1,080	3
$F_X(x)$	0	0,392	0,5	0,608	1

Dichtefunktion



Verteilungsfunktion



c) $E[X] \overset{(6.14)}{=} \int_0^1 (12x^3 - 12x^2 + 3x)\,dx = \left[\,3x^4 - 4x^3 + 1{,}5x^2\,\right]_0^1 = 3 - 4 + 1{,}5 = \mathbf{0{,}5}$ (Symmetrie!)

$V[X] \overset{(6.16)}{=} \int_0^1 (12x^4 - 12x^3 + 3x^2)\,dx - (0{,}5)^2 = \left[\,2{,}4x^5 - 3x^4 + x^3\,\right]_0^1 - 0{,}25 = 2{,}4 - 3 + 1 - 0{,}25 = \mathbf{0{,}15}$

d) $P(X \geq 0{,}4) \overset{(5.14)}{=} 1 - P(X < 0{,}4) \overset{(6.11)}{=} 1 - F_X(0{,}4) = 1 - (4 \cdot (0{,}4)^3 - 6 \cdot (0{,}4)^2 + 3 \cdot 0{,}4) = 1 - 0{,}496 = \mathbf{0{,}504}$

$P(|\,X - 0{,}5\,| \leq 0{,}3) = P(0{,}2 \leq X \leq 0{,}8) \overset{(6.12)}{=} F_X(0{,}8) - F_X(0{,}2) \overset{\textbf{b)}}{=} 0{,}608 - 0{,}392 = \mathbf{0{,}216}$

Aufgabe 6.4 (Fortsetzung)

e) Für die lineare Transformation $Y = c + d\,X$ gilt:

$$W_X = [0;1] \quad \text{und} \quad W_Y = [-1;1] \quad \Rightarrow \quad c + d \cdot 0 = -1 \quad \text{und} \quad c + d \cdot 1 = 1 \quad \Rightarrow \quad c = -1 \quad \text{und} \quad d = 2 \quad \text{d.h.} \quad \mathbf{Y = 2X - 1}$$

f) $$f_Y(y) \overset{(6.20)}{=} \frac{1}{|2|} f_X\left(\frac{y+1}{2}\right) \overset{a)}{=} \frac{12}{2}\left(\frac{y+1}{2}\right)^2 - \frac{12}{2}\left(\frac{y+1}{2}\right) + \frac{3}{2} = (1{,}5\,y^2 + 3\,y + 1{,}5) - (3y + 3) + 1{,}5 = \mathbf{1{,}5\,y^2} \qquad \text{für} \quad y \in [-1;1]$$

$$F_Y(y) \overset{(6.21)}{=} F_X\left(\frac{y+1}{2}\right) = 4\left(\frac{y+1}{2}\right)^3 - 6\left(\frac{y+1}{2}\right)^2 + 3\left(\frac{y+1}{2}\right) = (0{,}5\,y^3 + 1{,}5\,y^2 + 1{,}5\,y + 0{,}5) - (1{,}5\,y^2 + 3\,y + 1{,}5) + (1{,}5\,y + 1{,}5) = \mathbf{0{,}5\,y^3 + 0{,}5}$$

$$\text{für} \quad y \in [-1;1]$$

$$E[Y] \overset{(6.22)}{=} -1 + 2 \cdot E[X] \overset{\mathbf{c)}}{=} -1 + 2 \cdot 0{,}5 = \mathbf{0}$$

$$V[Y] \overset{(6.23)}{=} 2^2 \cdot V[X] \overset{\mathbf{c)}}{=} 4 \cdot 0{,}15 = \mathbf{0{,}6}$$